

Energy transfer between two filaments due to degenerate four-photon parametric process

Daniela A. Georgieva¹ and Lubomir M. Kovachev^{2,*}

¹ *Faculty of Applied Mathematics and Computer Science, Technical University of Sofia, 8, Kliment Ohridski Blvd., 1000 Sofia, Bulgaria*

² *Institute of Electronics, Bulgarian Academy of Sciences, Tzarigradsko shossee 72, 1784 Sofia, Bulgaria*

* *Corresponding author: lubomirkovach@yahoo.com*

Recently energy exchange between two filaments crossing at small angle and with power slightly above the critical for self-focusing P_{cr} was experimentally demonstrated. In this paper we present a model describing the process of this transfer through degenerate four-photon parametric mixing. Our model confirms the experimental results that the direction of energy exchange depends on the relative transverse velocity (incident angle), laser intensity and initial distance between the pulses (relative initial phase). We also investigate the interaction between two collinear filaments in order to explain the filaments number reduction for powers close to P_{cr} in multi-filamental propagation. © 2014 Optical Society of America

OCIS codes: 190.4380, 320.2250

1. Introduction

Several experimental works [1]- [3] report intensive exchange of energy when two filaments with the same carrying frequency intersect at a small angle. The mechanisms for energy exchange proposed in references [1] and [2] require two different main frequencies of the pulses. The experiment described in [3] is performed at a single pulse frequency. The authors of [3] add another mechanism as a possible explanation - nonlinear absorption at high input power. This process, though, cannot lead to a periodical energy exchange. All experiments were performed with filament intensities slightly above the critical value for self-focusing I_{cr} . The plasma density for laser pulses with such intensities is not high enough to provide the effect of plasma grating. The stability and the interaction of filaments with intensities

of the order of $I \sim 10^{12} \text{ W/cm}^2$ some authors try to explain as balance between Kerr self-focusing and ionization induced defocusing. In the book of P. Gibbon [4] the ionization induced defocusing is discussed widely. There are full coincidences between the numerical and experimental results, cited in the book. It is shown, that the ionization induced defocusing is possible only for intensities of the order of $I \sim 10^{15} \text{ W/cm}^2$, while in the experiments the measured intensity of a stable filament is of the order of $I \sim 10^{12} \text{ W/cm}^2$ - which is three orders of magnitude less. This is the reason to look for other mechanisms leading to exchange of energy and connected with nonlinear optical effects. To do that, we take into account the following experimental details. First, the laser pulses propagating on different optical trajectories always intersect with a relative phase difference. This means that at the intersection region there are conditions for degenerate four-photon parametric (FFP) processes. As is mentioned in [5], the degenerate FFP processes play an important role also in the multi-component filamentation generation. Another important experimental observation in the multi-filament propagation is the significant reduction in the number of filaments as a function of the distance [6–8]. Up to now we have not seen a physical model explaining this phenomenon.

In this manuscript we propose a nonlinear vector model including degenerate FFP process. We investigate numerically the interaction between optical pulses in the cases when 1) the components of the polarization vector are separated, 2) the filaments contain the two polarization components (the polarization components are not separated, general case). The phenomenon of the reduction of the number of filaments during multi-filament propagation is also studied in our model. Our analysis shows that the decreasing of the number of filaments could be due to pairing of the filaments and energy exchange due to degenerate FFP mixing. The numerical scheme is performed on the base of the split-step Fourier method.

2. Nonlinear Polarization

As demonstrated in [9], high intensity femtosecond pulses generate in gases stable filaments with broad-band spectrum. Various models explaining this phenomenon are presented (see for example the paper of Couairon and Mysyrowicz and references therein [7]). The standard filamentation model is based on plasma generation and multi-photon processes and includes also nonlinear polarization of the kind

$$\vec{P}^{nl} = n_2 \left[(\vec{E} \cdot \vec{E}^*) \vec{E} + \frac{1}{2} (\vec{E} \cdot \vec{E}) \vec{E}^* \right], \quad (1)$$

where n_2 is the nonlinear refractive index of the isotropic media. The polarization (1) was proposed by Maker and Terhune in 1965 [10]. If the electrical field contains one linear or circular component, the polarization (1) describes only the self-action effect, while in the case

of two-component electrical field $\vec{E} = (E_x, E_y, 0)$ additional terms appear, presenting cross-modulation and degenerate four-photon parametric mixing. The self-action process broadens the pulse spectrum - starting from narrow-band pulse, the stable filament becomes broad-band far way from the source. Later [11, 12] show, though, that the evolution of broad-band pulses like filaments can not be described correctly by nonlinear polarization of the kind (1). It is more correct to use the generalized nonlinear operator

$$\vec{P}^{nl} = n_2 (\vec{E} \cdot \vec{E}) \vec{E}, \quad (2)$$

which includes additional processes associated with third harmonic generation (THG). The more precise analysis, presented in the present paper, demonstrates that the polarization of kind (2) is not applicable to a scalar model, because the corresponding Manley-Rowe (MR) conservation laws are not satisfied. That is why we substitute into the nonlinear operators (1) and (2) a two-component electrical vector field at one carrying frequency of the form:

$$\vec{E} = \frac{(A_x \exp(i\omega_0 t) + c.c.)}{2} \vec{x} + \frac{(A_y \exp(i\omega_0 t) + c.c.)}{2} \vec{y}, \quad (3)$$

where $A_x = A_x(x, y, z, t)$, $A_y = A_y(x, y, z, t)$ are the amplitude functions and ω_0 is the carrying frequency of the laser source.

In the case of Maker and Terhune polarization (1) we obtain

$$\begin{aligned} \vec{P}_x^{nl} &= \frac{3}{8} n_2 \left[\left(|A_x|^2 + \frac{2}{3} |A_y|^2 \right) A_x + \frac{1}{3} A_x^* A_y^2 \right] \exp(i\omega_0 t) + c.c. \\ \vec{P}_y^{nl} &= \frac{3}{8} n_2 \left[\left(|A_y|^2 + \frac{2}{3} |A_x|^2 \right) A_y + \frac{1}{3} A_y^* A_x^2 \right] \exp(i\omega_0 t) + c.c. \end{aligned} \quad (4)$$

The nonrestricted nonlinear polarization (2) generates the following components

$$\begin{aligned} \vec{P}_x^{nl} &= \frac{3}{8} n_2 \left[\frac{1}{3} (A_x^2 + A_y^2) A_x \exp(2i\omega_0 t) + \left(|A_x|^2 + \frac{2}{3} |A_y|^2 \right) A_x + \frac{1}{3} A_x^* A_y^2 \right] \exp(i\omega_0 t) + c.c. \\ \vec{P}_y^{nl} &= \frac{3}{8} n_2 \left[\frac{1}{3} (A_x^2 + A_y^2) A_y \exp(2i\omega_0 t) + \left(|A_y|^2 + \frac{2}{3} |A_x|^2 \right) A_y + \frac{1}{3} A_y^* A_x^2 \right] \exp(i\omega_0 t) + c.c. \end{aligned} \quad (5)$$

Comparing (4) and (5), it is clearly seen that the operator (2) $n_2 (\vec{E} \cdot \vec{E}) \vec{E}$ generalizes the case of Marker and Terhune's operator (1), but includes also additional terms associated with THG.

3. Basic System of Equations

The stable filament propagation in gases is realized in the sub-pico and femtosecond regions, while in the nano- and picosecond regions appears the well known self-focusing. The dynamics of narrow-band laser pulses can be accurately described in the frame of paraxial optics. The filamentation experiments demonstrate a typical pulse spectrum evolution. The initial laser pulse ($t_0 \geq 50fs$) possesses a relatively narrow-band spectrum ($\Delta k_z \ll k_0$), where Δk_z is the spectral pulse width and k_0 is the carrying wave number. During the filamentation process the initial self-focusing broadens significantly the pulse spectrum. The broad-band spectrum ($\Delta k_z \sim k_0$) is one of the basic characteristics of the stable filament. The evolution of the so obtained filament can not be further described in the frame of the nonlinear paraxial optics because the paraxial optics works correctly for narrow-band laser pulses only. The dynamics of broad-band pulses can be presented properly within different non-paraxial models such as UPPE [11] or non-paraxial envelope equations [14]. Another standard restriction in the filamentation theory is the use of one-component scalar approximation of the electrical field \vec{E} . This approximation though, is in contradiction with recent experimental results, where rotation of the polarization vector is observed [13]. For this reason, in the present paper we use the non-paraxial vector model up to second order of dispersion, in which the nonlinear effects are described by the nonlinear polarization components (5). The system of non-paraxial equations of the amplitude functions A_x, A_y of the two-component electrical field (3) has the form

$$\begin{aligned}
-2i \frac{k_0}{v_{gr}} \frac{\partial A_x}{\partial t} &= \Delta_{\perp} A_x - \frac{\beta + 1}{v_{gr}} \left(\frac{\partial^2 A_x}{\partial t^2} - 2v_{gr} \frac{\partial^2 A_x}{\partial t \partial z} \right) - \beta \frac{\partial^2 A_x}{\partial z^2} \\
&+ k_0^2 \tilde{n}_2 \left[\frac{1}{3} (A_x^2 + A_y^2) A_x \exp(2ik_0(z - (v_{ph} - v_{gr})t)) + \left(|A_x|^2 + \frac{2}{3}|A_y|^2 \right) A_x + \frac{1}{3} A_x^* A_y^2 \right] \\
-2i \frac{k_0}{v_{gr}} \frac{\partial A_y}{\partial t} &= \Delta_{\perp} A_y - \frac{\beta + 1}{v_{gr}} \left(\frac{\partial^2 A_y}{\partial t^2} - 2v_{gr} \frac{\partial^2 A_y}{\partial t \partial z} \right) - \beta \frac{\partial^2 A_y}{\partial z^2} \\
&+ k_0^2 \tilde{n}_2 \left[\frac{1}{3} (A_x^2 + A_y^2) A_y \exp(2ik_0(z - (v_{ph} - v_{gr})t)) + \left(|A_y|^2 + \frac{2}{3}|A_x|^2 \right) A_y + \frac{1}{3} A_y^* A_x^2 \right],
\end{aligned} \tag{6}$$

where $\tilde{n}_2 = \frac{3}{8}n_2$, v_{gr} and v_{ph} are the group and phase velocities correspondingly, $\beta = k_0 v_{gr}^2 k''$ and k'' is the group velocity dispersion.

This model describes the ionization-free filamentation regime, where the pulse intensities are close to the critical one for self-focusing. The first nonlinear term in (6) corresponds to coherent GHz generation [12]. The system (6) is written in Galilean frame ($z' = z - vt; t' = t$) and not in the standard local time frame ($t' = t - z/v; z' = z$). In all coordinate systems - laboratory, moving in time, and Galilean, the group velocity adds an additional phase

(carrier-envelope phase) in the third harmonic terms and transforms them to GHz ones. This can be seen directly for the system (6) written in Galilean frame, which determines the choice of coordinates. The last nonlinear term in (6) describes degenerate four-photon parametric mixing (FPM).

The system of equations (6) written in dimensionless form becomes

$$\begin{aligned}
& -2i\alpha\delta^2\frac{\partial A_x}{\partial t} = \Delta_{\perp}A_x - \delta^2(\beta+1)\left(\frac{\partial^2 A_x}{\partial t^2} - \frac{\partial^2 A_x}{\partial t\partial z}\right) - \delta^2\beta\frac{\partial^2 A_x}{\partial z^2} \\
& +\gamma\left[\frac{1}{3}\left(A_x^2 + A_y^2\right)A_x \exp(2i(\alpha z - \tilde{\omega}_{nl}t)) + \left(|A_x|^2 + \frac{2}{3}|A_y|^2\right)A_x + \frac{1}{3}A_x^*A_y^2\right] \\
& -2i\alpha\delta^2\frac{\partial A_y}{\partial t} = \Delta_{\perp}A_y - \delta^2(\beta+1)\left(\frac{\partial^2 A_y}{\partial t^2} - \frac{\partial^2 A_y}{\partial t\partial z}\right) - \delta^2\beta\frac{\partial^2 A_y}{\partial z^2} \\
& +\gamma\left[\frac{1}{3}\left(A_x^2 + A_y^2\right)A_y \exp(2i(\alpha z - \tilde{\omega}_{nl}t)) + \left(|A_y|^2 + \frac{2}{3}|A_x|^2\right)A_y + \frac{1}{3}A_y^*A_x^2\right],
\end{aligned} \tag{7}$$

where $x = x/r_0$, $y = y/r_0$, $z = z/r_0$ are the dimensionless coordinates, r_0 is the pulse waist, $z_0 = v_{gr}t_0$ is the spatial pulse length, $\alpha = k_0z_0$, $\delta = r_0/z_0$, $\gamma = k_0^2r_0^2\tilde{n}_2|A_0|^2/2$ is the nonlinear coefficient and $\tilde{\omega}_{nl} = k_0(z - (v_{ph} - v_{gr}))t_0$ is the normalized nonlinear frequency.

4. Conservation Laws

The nonlinear theories based on the polarization of Maker and Terhune type (1) satisfy the Manley-Rowe relations. This means that during the process of energy exchange the total energy is conserved for arbitrary localized smooth complex fields. Additional conservation quantities are also possible. To satisfy the MR relations of the truncated equations with a generalized nonlinear polarization of type $\vec{P}^{nl} = n_2(\vec{E} \cdot \vec{E})\vec{E}$, some restrictions on the components of the electrical field are imposed. We will demonstrate this on the basis of two-component vector field $\vec{E} = (A_x, A_y, 0)\exp[ik_0(z - v_{ph}t)]$. Let us rewrite the generalized nonlinear polarization (2) using circularly polarized components [5, 15]

$$A_+ = (A_x + iA_y)/\sqrt{2}, \quad A_- = (A_x - iA_y)/\sqrt{2}. \tag{8}$$

Thus we obtain

$$P_+ = n_2(A_+^2A_-)\exp[ik_0(z - v_{ph}t)] \tag{9}$$

$$P_- = n_2(A_-^2A_+)\exp[ik_0(z - v_{ph}t)], \tag{10}$$

where by convention P_+ and P_- correspond to left-hand circular and to right-hand circular polarization. The truncated equations can be written as

$$i\frac{\partial A_+}{\partial t} = n_2 (A_+^2 A_-) \quad (11)$$

$$i\frac{\partial A_-}{\partial t} = n_2 (A_-^2 A_+). \quad (12)$$

The equations for the square modulus of the components A_+ and A_- are

$$i\frac{\partial |A_+|^2}{\partial t} = n_2 |A_+|^2 (A_+ A_- - A_+^* A_-^*) = 0 \quad (13)$$

$$i\frac{\partial |A_-|^2}{\partial t} = n_2 |A_-|^2 (A_+ A_- - A_+^* A_-^*) = 0,$$

following from the fact that the components A_+ and A_- are complex-conjugated fields (8). Thus we prove that to satisfy the MR conditions for the nonlinear system (7) (or other conservative nonlinear equations) with nonlinear operator of kind (2), the possible initial conditions and solutions should be complex-conjugated fields. The conservation laws (13) give us additional information on the behavior of the vector amplitude function: *only components of the vector amplitude field $\vec{A} = (A_x, A_y, 0)$, which present rotation of the vector \vec{A} in the plane (x, y) , satisfy the MR conditions.* That is why in our numerical experiments, as well as in our analytical investigations, we will use complex-conjugated components only.

5. Numerical simulations and discussions

We investigate numerically the following two basic nonlinear effects in the filamentation process - the energy exchange between two non-collinear filaments and the phenomenon of reducing the number of filaments in multi-filamentation propagation. We demonstrate that both processes are based on degenerate FPM mixing. The numerical simulations are carried out by using the split-step Fourier method. The numerical results are presented for initial conditions: 240fs Gaussian bullet with waist and spatial length $r_0 = z_0 = 72\mu m$ and power slightly above P_{cr} . In this case $\alpha = 200\pi$, $\delta = 1$, $\tilde{\omega}_{nl} = 0.00023$ and $\gamma \in 1.5 - 3$. The phase difference between the two components is initially $\pi/2$ in order to satisfy the conservation laws.

5.A. Energy Exchange between Two Non-collinear Filaments

Let us begin our consideration with the case when two laser pulses with the same carrying frequency intersect at a small angle. The pulse power is slightly above the critical for self-focusing P_{cr} .

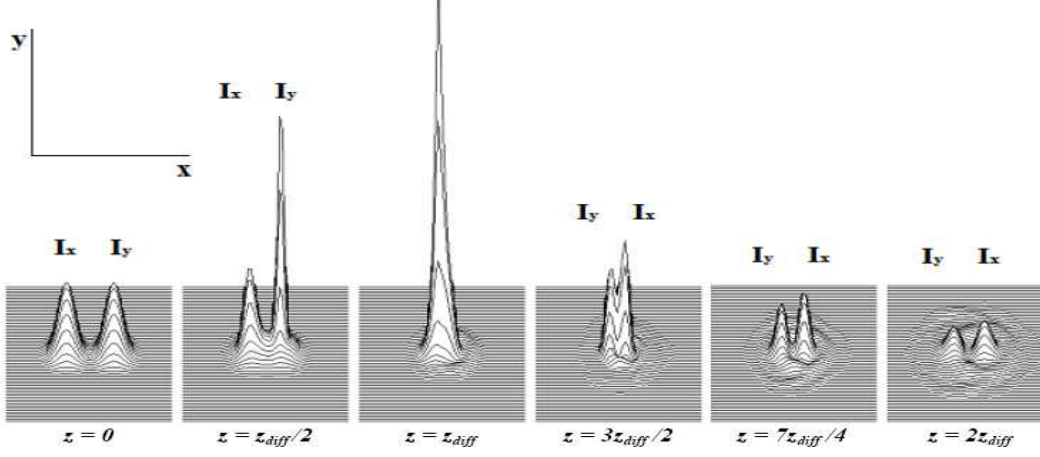


Fig. 1. Collision dynamics between two separated polarization components at initial distance $2a = 4$, transverse velocity difference $2\Delta v = 3$, and nonlinear coefficient $\gamma = 1, 5$, governed by the system of equations (7) with initial conditions (14). In the process of collision we observe self-focusing and periodic exchange of energy. The initial energy transfer is from I_x to I_y . With z_{diff} is denoted the diffraction lengths $z_{diff} = k_0 r_0^2$.

5.A.1. Polarization Separated Components

The separation of the polarization components can be realized experimentally by means of a system of prisms, where the light pulse falls at Brewster angle. In this case the initial conditions for numerical solution of the system of equations (7) have the form

$$\begin{aligned}
 A_x &= A_x^0 \exp \left(-\frac{(x+a)^2 + y^2 + z^2}{2} \right) \exp(-i\Delta v x) \\
 A_y &= A_y^0 \exp \left(-\frac{(x-a)^2 + y^2 + z^2}{2} \right) \exp(i\Delta v x) \exp \left(i\frac{\pi}{2} \right),
 \end{aligned} \tag{14}$$

where a is the initial shift of the pulses in x -direction with respect to the intersection point and $2\Delta v = 2v \sin \theta$ is the normalized relative transverse velocity of the pulses (θ is the angle between the two trajectories).

The crossing of the optical pulses A_x and A_y for $\gamma = 1.5$, $\Delta v = 1.5$ and $a = 2$ is presented in Fig. 1. With $I_x = |A_x|^2$ and $I_y = |A_y|^2$ are denoted the intensity profiles of the spots (x, y) projections of the pulses, while z_{diff} denotes the diffraction lengths $z_{diff} = k_0 r_0^2$. The behavior of the intensity profiles I_x and I_y separately is shown in Fig. 2. The intensive periodical energy exchange and simultaneous self-focusing of A_x and A_y is evident. The

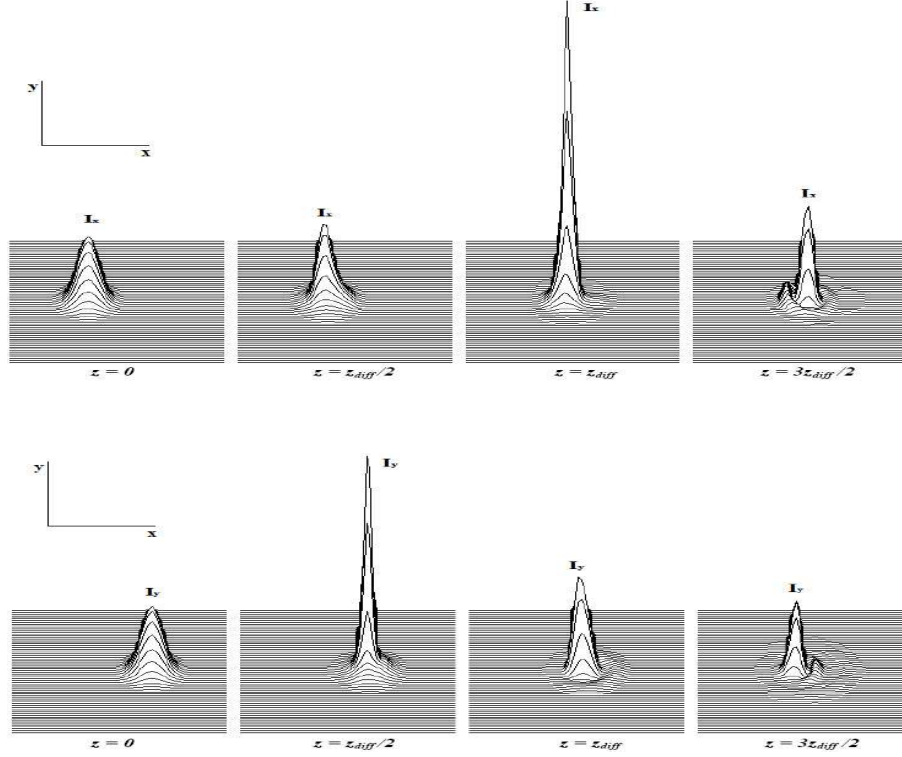


Fig. 2. Evolution of the intensity profile I_x of the A_x -component and the intensity profile I_y of the A_y -component of the vector field \vec{E} (presented separately) for the same initial conditions as in Fig. 1. The periodicity of the energy exchange is clearly seen.

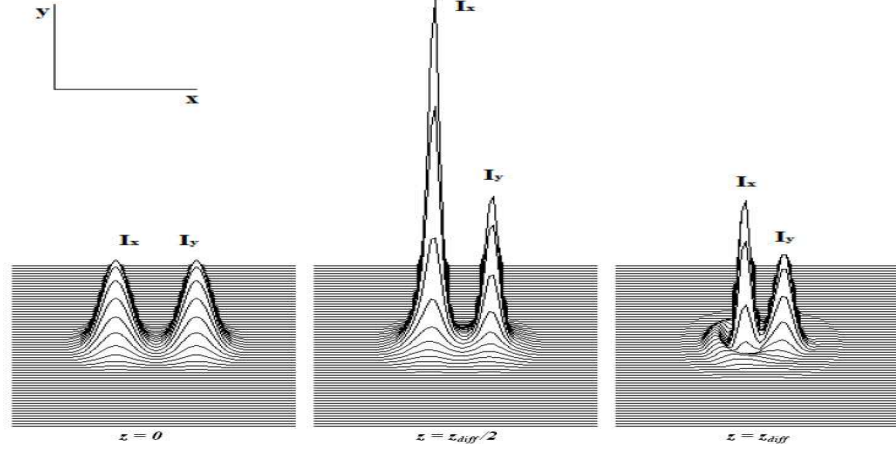


Fig. 3. Crossing dynamics between two separated polarization components at initial distance $2a = 4$, transverse velocity difference $2\Delta v = 2$, and nonlinear coefficient $\gamma = 1, 5$, governed by the system of equations (7) with initial conditions (14). The value of the parameter $2\Delta v$ changes the initial phase difference. As a result, the direction of energy transfer is changed from I_y to I_x .

periodicity of the FFP process can be seen very well in Fig. 2. In this case the initial energy transfer is from I_x to I_y .

In our study the magnitude and direction of energy transfer depend on the initial shift of the pulses a , the relative transverse velocity Δv and the laser intensity γ . Fig. 3 presents a similar numerical simulation as Fig. 1, with $\Delta v = 1$ (instead $\Delta v = 1.5$). In this experiment the direction of energy transfer is changed from I_y to I_x . This confirms the experimental results obtained in [3], where similar dependencies of the transfer energy direction on the relative time delay, laser intensities and intersecting angle ($\Delta v = v \sin \theta$) are presented.

5.A.2. Non-separated Polarization Components

When the laser pulse is split into two arms \vec{A}_1 and \vec{A}_2 by a regular beam splitter, the obtained filaments contain each of the polarization components. Let $\vec{A}_j = A_{j,x}\vec{x} + A_{j,y}\vec{y}$, $j = 1, 2$. Then the initial conditions for numerical solution of the system of equations (7) have the form

$$\begin{aligned}
 A_x = A_{1,x} + A_{2,x} = & \frac{A_1^0}{\sqrt{2}} \exp\left(-\frac{(x+a)^2 + y^2 + z^2}{2}\right) \exp(-i\Delta vx) \\
 & + \frac{A_2^0}{\sqrt{2}} \exp\left(-\frac{(x-a)^2 + y^2 + z^2}{2}\right) \exp(i\Delta vx) \exp\left(i\frac{\pi}{2}\right)
 \end{aligned} \tag{15}$$

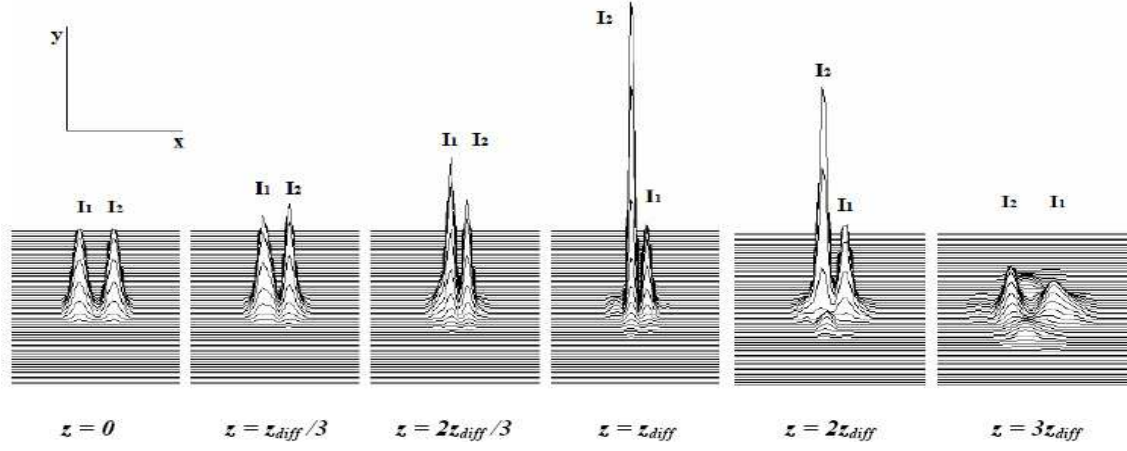


Fig. 4. Evolution of two filaments with non-separated polarization components at initial distance $2a = 4$, transverse velocity difference $2\Delta v = 3$, and nonlinear coefficient $\gamma = 1, 5$, governed by the system of equations (7) with initial conditions (15). We observe self-focusing, periodic exchange of energy and in the faraway zone - generation of two additional components in direction orthogonal to the initial pulses.

$$A_y = A_{1,y} + A_{2,y} = \left\{ \frac{A_1^0}{\sqrt{2}} \exp \left(-\frac{(x+a)^2 + y^2 + z^2}{2} \right) \exp(-i\Delta vx) + \frac{A_2^0}{\sqrt{2}} \exp \left(-\frac{(x-a)^2 + y^2 + z^2}{2} \right) \exp(i\Delta vx) \exp \left(i\frac{\pi}{2} \right) \right\} \exp \left(i\frac{\pi}{2} \right),$$

where A_x and A_y are composed of the x - and y -components of the two optical pulses propagating along different trajectories.

The interaction of the optical pulses \vec{A}_1 and \vec{A}_2 for $\gamma = 1.5$, $\Delta v = 1.5$ and $a = 2$ is shown in Fig. 4. As in the previous case of separated polarization components we observe the effects of self-focusing and intensive periodical energy exchange. In the faraway zone two additional components in direction orthogonal to the initial pulses are obtained. We suppose that these additional components are connected with the four-wave mixing process. Recently similar experimental results have been reported in [16].

5.B. Energy Exchange between Two Collinear Filaments

The propagation through atmosphere of high intensity laser pulse with power two orders of magnitude greater than the critical for self-focusing ($P \sim 100P_{cr}$) leads to breakup of the pulse into many components, each with power around P_{cr} [5]. The basic idea is that filamentation occurs as a consequence of initially present laser wavefront irregularities, enhanced

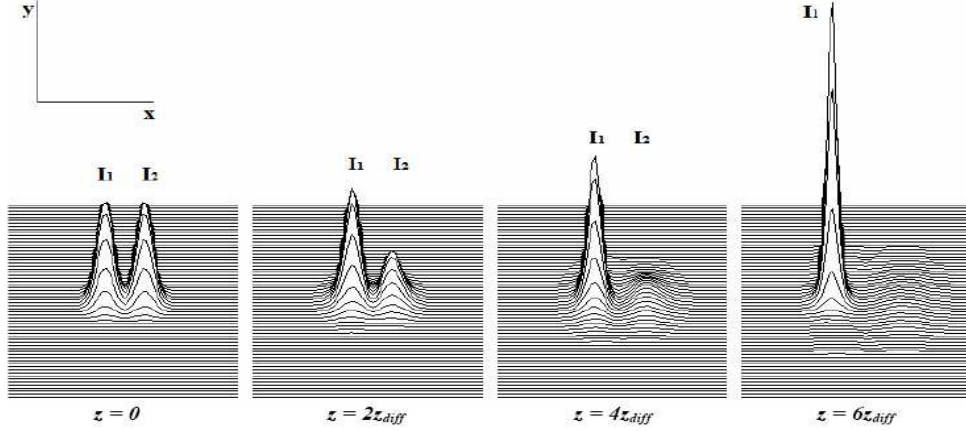


Fig. 5. Energy exchange between two collinear filaments \vec{A}_1 and \vec{A}_2 at small a distance $2a = 3.4$ governed by the system of equations (7) with initial conditions (15) at values of the parameters $v = 0.00001$ and $\gamma = 1.5$. Due to degenerated FFP mixing one of the filaments is amplified while the other filament enters in linear mode and vanishes.

by four-wave mixing. As was reported recently [8], the number of filaments is reduced significantly as a function of the distance. We propose a model based on the degenerate FFP process in order to explain this decreasing number of filaments. Let us suppose that two filaments are at a small distance from each other. Therefore there are conditions for pairing and interaction due to degenerate FFP mixing. The formulation of the problem is similar to the last case in the previous subsection. Let us consider two filaments \vec{A}_1 and \vec{A}_2 with arbitrary polarization. Let us write the vectors through their x - and y -components $\vec{A}_j = A_{j,x}\vec{x} + A_{j,y}\vec{y}$, $j = 1, 2$. Then the initial conditions for the numerical solution of the system (7) have the form (14), with relative velocity $\Delta v = 0.00001$ (hence $\theta \sim 0$, i.e. the filaments are collinear).

Fig 5. shows the evolution of two filaments and their exchange of energy by degenerate FFP mixing. We use power close to P_{cr} . Therefore the amplified pulse self-focuses and gets enough power to continue its propagation, while the other pulse gives out energy, enters into linear mode and vanishes. In this way the number of filaments can be reduced by non-linear parametric processes in $\chi^{(3)}$ media.

6. Conclusions

We have developed a model to describe the recent experimental demonstration of energy exchange between two non-collinear filaments in air, with power close to P_{cr} [1]- [3]. The

authors there point out a possible mechanism based on the generation of plasma grating at the interaction point. It is known though, that at higher incident laser power, where the plasma plays an important role, the exchange efficiency decreases. In air $P = P_{cr}$ corresponds to intensity of the laser field of the order of $I \sim 10^{12} \text{ W/cm}^2$, where the plasma density is too small to form plasma grating.

In this paper we propose another mechanism for this phenomenon on the basis of degenerate FFP mixing. Our numerical investigation confirms the experimental results of periodical energy exchange in the region of overlapping of the two pulses. We obtain also the observed in the experiments dependence of the initial energy transfer on the relative transverse velocity (crossing angle), intensity and initial phase difference (distance a). When the pulse A_1 has positive initial phase with respect to pulse A_2 (depending on the above parameters), initially pulse A_1 takes energy from pulse A_2 and vice versa. As pointed out in [5], the degenerate FFP processes play an important role in the multi-filament generation. Another important effect, which was observed experimentally in the multi-filament propagation, is that the number of filaments N with power around P_{cr} decreases gradually with propagation distance [6–8]. The authors explain this with energy losses by plasma or multi-photon absorption. We think, though, that for intensities of the order of $I \sim 10^{12} \text{ W/cm}^2$ the multi-photon processes and plasma are very weak to play such important role in this process. That is why we turn back to the theory developed by Boyd [5], and we extend it now to evolution equations for pairing of pulses by degenerate FFP process. The result is that one of the pulses is amplified, while the other one decreases in energy, enters in linear mode and vanishes. Thus, the degenerate FFP process is a natural explanation of the reducing of the number of filaments with power around P_{cr} . Another important result in this study is that we investigate the filament and the filament's interaction as a vector field. Beyond the scope of the specific task, we have shown that the vector representation of the problems removes the disadvantages of the use of the generalized nonlinear polarization operator (2), making it a useful tool in the solution of different problems, connected with third-order polarization. We plan to use in the future this (or a similar) vector system for description of the observed in [16] FFP vector solitons.

7. ACKNOWLEDGMENTS

This work was supported in part by Technical University of Sofia under grant No.141PD0001 – 11 and grant DDVU02/71 with the Bulgarian Science Fund.

References

1. A. C. Bernstein, M. McCormick, G. M. Dyer, J. C. Sanders, and T. Ditmire, "Two-beam coupling between filament-forming beams in air," *Phys. Rev. Lett.* **102**, 123902 (2009).

2. Y. Liu, M. Durand, S. Chen, A. Houard, B. Prade, B. Forestier, and A. Mysyrowicz, "Energy exchange between femtosecond laser filaments in air," *Phys. Rev. Lett.* **105**, 055003 (2010).
3. Pengji Ding, Zeqing Guo, Xiaoshan Wang, Yu Cao, Mingze Sun, Peixi Zhao, Yanchao Shi, Shaohua Sun, Xiaoliang Liu, and Bitao Hu, "Energy exchange between two noncollinear filament-forming laser pulses in air", *Optics Express*, **21**, 27631-27641 (2013).
4. P. Gibbon *Short Pulse Laser Interaction With Matter* (Imperial College Press, 2005, pp. 24-27)
5. R. W. Boyd *Nonlinear Optics*, (Academic Press, 2003).
6. S. L. Chin and all , "The propagation of powerfull femtosecond laser pulses in optical media: physics, applications and new challenges", *Can. J. Phys.* **83**, 863-905 (2005).
7. A. Couairon, and A. Mysyrowicz, "Femtosecond filamentation in transparent media", *Physics Reports*, **441**, 47-189 (2007).
8. Magali Durand and all, "Kilometer range filamentation", *Optics Express*, **21**, 26836 (2013).
9. A. Braun, G. Korn, X. Liu, D. Du, J. Squier, and G. Mourou, "Self-channeling of high-peak-powerfemtosecond laser pulses in air", *Opt. Lett.*, **20**(1), 73-75 (1995).
10. P. D. Maker and R. W. Terhune, "Study of Optical Effects Due to an Induced Polarization Third Order in the Electric Field Strength" *Phys. Rev.* **137**, A801 (1965).
11. M. Kolesik, J. V. Moloney, "Perturbative and non-perturbative aspects of optical filamentation in bulk dielectric media", *Optics Express*, **16**, 2971 (2008), M. Kolesik, E. M. Wright, A. Becker, J. V. Moloney, "Simulation of third-harmonic and supercontinuum generation for femtosecond pulses in air", *Appl. Phys. B*, 85, pp 531-538 (2006).
12. L. M. Kovachev, "New mechanism for THz oscillation of the nonlinear refractive index in air: particle-like solutions", *Journal of Modern Optics*, **56**, 1797- 1803 (2009).
13. A. H. Sheinfux, E. Schleifer, J. Papeer, G. Gibich, B. Ilan, andA. Zigler, "Measuring the stability of polarization orientation in high intensity laser filaments in air.", *Applied Physics Letters* **101**, 201105 (2012).
14. Lubomir M. Kovachev and Kamen Kovachev, "Linear and Nonlinear Femtosecond Optics in Isotropic Media. Ionization-free Filamentation", *Laser Pulses / Book 1*, chapter, InTech, 2011.
15. G. P. Agrawal, *Nonlinear Fiber Optics*, 4th ed. (Academic, 2007).
16. Ruimin Wang and all, "Observation of multi-component spatial vector solitons of four-wave mixing", *Optics Express*, **20**, 14168 (2012).